INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 39, Northern Spring 2018 (A Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Thirty nine non-zero numbers are written in a row. The sum of any two neighbouring numbers is positive, while the sum of all the numbers is negative. Is the product of all these numbers negative or positive? (4 points)

2. Aladdin has several gold coins and from time to time he asks the Genie to give him more. On each such occasion the Genie first responds by adding a thousand gold coins and then he takes back a half of the total weight of all Aladdin's gold coins. If after asking the Genie for more gold ten times, is it possible for Aladdin that the number of his gold coins has increased taking into account that each time the Genie takes a half of all Aladdin's gold back and no coin is broken into smaller pieces? (5 points)

3. Do there exist 2018 positive reduced fractions, each with a different denominator, such that the denominator of the difference of any two (after reducing to lowest terms) is less than the denominator of any of the initial 2018 fractions? (6 points)

4. Let O be the circumcentre of triangle ABC. Let AH be an altitude of triangle ABC, and let P be the foot of the perpendicular dropped to the line CO from point A. Prove that the line HP passes through the midpoint of the side AB. (6 points)

5. There are 100 houses in a street, which are divided into 50 pairs. Each pair are located opposite one another in the street. On the right side of the street all houses have even numbers, while all houses on the left side have odd numbers. On both sides of the street the numbers increase from one end of the street to the other, but the numbers are not necessarily consecutive (some numbers may be skipped). For each house on the right side of the street, the difference between its number and the number of the opposite house is calculated, and as it turns out, all the differences are distinct from one another. Let n be the greatest number of a house on the street. Find the smallest possible value of n. (8 points)

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6. In the land of knights (who always tell the truth) and knaves (who always lie), 10 people, at least one of them a knave, sit at a round table, each at a vertex of an inscribed regular 10-gon. A traveller can choose to stand at any point outside the table and ask the people at the table:

"What is the distance from me to the nearest knave at the table?"

After that each person at the table gives him an answer. What is the minimal number of questions the traveler has to ask to determine for sure what people at the table are knaves? (The people at the table and the traveller are to be considered as points, and everyone, including the traveller, can make exact measurement of the distance between any two points.) (10 points)

7. You are travelling to some country and you don't know its language. You know that symbols "!" and "?" stand for addition and subtraction, but you don't know which symbol is for which operation. Each of these two symbols can be written between two arguments, but for subtraction you don't know if the left argument is subtracted from the right or vice versa. For example, $a?b$ could mean any of $a - b$, $b - a$ and $a + b$. You don't know how to write any numbers, but variables and brackets can be used as usual. Given two arguments a and b how can you write for sure an expression that is equal to $20a - 18b$? (12 points)